



**P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA**  
**SEM END EXAMINATIONS NOV -2024**  
**II BSC CHEMISTRY/STATISTICS MINOR : GROUP THEORY**

|                |               |        |  |  |  |  |  |  |  |  |           |    |
|----------------|---------------|--------|--|--|--|--|--|--|--|--|-----------|----|
| DATE & SESSION | 25.11.24 & AN | REG NO |  |  |  |  |  |  |  |  | MAX MARKS | 50 |
|                |               |        |  |  |  |  |  |  |  |  |           |    |

TIME: 2 HRS

Answer any three questions. Selecting at least one question from each part

**Part – I**

3 X 10 = 30

1. Show that the set  $Q_+$  of all positive rational numbers forms an abelian group under the composition defined by ' $\circ$ ' such that  $a \circ b = (ab)/3$  for  $a, b \in Q_+$ .
2. State and prove Lagrange's Theorem. Prove that the converse of Lagrange's theorem is not true.
3. Prove that  $H$  is a normal sub-group of  $G$  if and only if product of two right (left) cosets of  $H$  in  $G$  is again a right (left) coset of  $H$  on  $G$ .

**Part – II**

4. State and prove fundamental theorem of homomorphism of groups.
5. State and prove Cayley's theorem
6. Prove that every subgroup of a cyclic group is cyclic.

**SECTION-B**

Answer any four questions

4 X 5 M = 20 M

7. If  $G$  is a group, for  $a, b \in G$  prove that  $(ab)^{-1} = b^{-1}a^{-1}$
8. Prove that the group  $(G, \bullet)$  is abelian iff  $(ab)^2 = a^2b^2$
9. If  $H$  and  $K$  are two subgroups of a group  $G$  then show that  $H \cap K$  is also a subgroup of  $G$ .
10. If  $H$  is a subgroup of a group  $G$  then show that  $HH = H$ .
11. Prove that every subgroup of an abelian group is normal.
12. Prove that the homomorphic image of an abelian group is abelian.
13. Verify whether the permutation  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 5 & 4 & 3 & 6 & 1 & 7 & 9 & 8 \end{pmatrix}$  is even or odd.